EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

Prof. Dr. Andreas Prohl
Benita Tscheppe

Nichtlineare Optimierung

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Übungsaufgaben 10

Problem 1. Consider the linear program in normal form, with data

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 4 \\ 6 \\ 2 \\ 3 \end{pmatrix}, \qquad \mathbf{c}^\top = (-2, -3, -4, 0, 0, 0, 0)\,.$$

Do one simplex step, starting with the corner $\mathbf{x} = (2, 0, 0, 2, 6, 0, 3)^\top$.

Problem 2. The aim is to approximate the following optimization problem with equality constraints

$$\min f(\mathbf{x}) \qquad \text{subject to} \qquad h_j(\mathbf{x}) = 0 \quad (1 \leq j \leq p) \tag{1}$$

by the (un-constrained) penalty method (with penalty parameter $\gamma > 0$)

$$\min_{\mathbf{x} \in \mathbb{R}^n} \mathscr{P}(\mathbf{x}; \gamma)\,, \qquad \text{where} \qquad \mathscr{P}(\mathbf{x}; \gamma) := f(\mathbf{x}) + \frac{\gamma}{2} \|\mathbf{h}(\mathbf{x})\|^2_{\mathbb{R}^p}\,. \tag{2}$$

In the lecture, we had a theorem which stated properties for the penalty method, under the assumptions that $\mathscr{X} := \mathbf{h}^{-1}(\mathbf{0}) \neq \emptyset$, that $\{\gamma_k\}_k \subset \mathbb{R}^+$ would be strictly growing, that $f, \mathbf{h}$ are continuous, and f would be bounded from below, as well as that (2) has a unique solution $\mathbf{x}^*_k$ for every $k \in \mathbb{N}$.

In the lecture, we verified already that the sequence $\{\mathscr{P}(\mathbf{x}^*_k; \gamma_k)\}_k$ is growing, and that $\{\|\mathbf{h}(\mathbf{x}^*_k)\|_{\mathbb{R}^p}\}_k$ is decreasing.

Show that the following properties are valid:

i) The sequence $\{f(\mathbf{x}^*_k)\}_k$ is growing.

ii) $\mathbf{h}(\mathbf{x}^*_k) \to \mathbf{0} \quad (k \uparrow \infty)$.

iii) Every accumulation point of the sequence $\{\mathbf{x}^*_k\}_k \subset \mathbb{R}^n$ solves (1).

Problem 3. In addition to the data assumptions in **Problem 2**, let us assume that $f, \mathbf{h}$ are both continuously differentiable, and that the limit $\mathbf{x}^* := \lim_{k\uparrow\infty} \mathbf{x}^*_k$ is a *regular* point. Then the approximates $\{\boldsymbol{\lambda}^k\}_k \subset \mathbb{R}^p$, with $\lambda^k_j = \gamma_k h_j(\mathbf{x}^*_k)$, converge to the unique Lagrange multiplier $\boldsymbol{\lambda} \in \mathbb{R}^p$ of (1).

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