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Fachbereich Mathematik

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Nichtlineare Optimierung

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Übungsaufgaben 9

Problem 1. Consider the optimization problem:

$$\min\ x_1 + x_2 \qquad \text{subject to} \qquad \begin{cases} x_1 + x_2 \geq 1 \\ x_1 \geq 0,\ x_2 \geq 0 \end{cases}$$

i) Give the solutions to this problem.

ii) Please state the problem as LP in normal form.

Problem 2. Let $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, let $\mathbf{c} \in \mathbb{R}^n$, and $\mathbf{b} \in \mathbb{R}^m$. Consider the (primal) linear program

$$\min(\mathbf{c}, \mathbf{x}) \qquad \text{subject to} \qquad \boldsymbol{A}\mathbf{x} = \mathbf{b} \quad \text{and} \quad \mathbf{x} \geq \mathbf{0}\,,$$

as well as the related *dual problem*

$$\max(\mathbf{b}, \boldsymbol{\lambda}) \qquad \text{subject to} \qquad \boldsymbol{A}^\top\boldsymbol{\lambda} + \mathbf{s} = \mathbf{c} \quad \text{and} \quad \mathbf{s} \geq \mathbf{0}\,.$$

i) Please detail that the conditions

$$\begin{aligned} \boldsymbol{A}^\top\boldsymbol{\lambda} + \mathbf{s} &= \mathbf{c} \\ \boldsymbol{A}\mathbf{x} &= \mathbf{b} \\ x_i \geq 0,\ s_i \geq 0,\ x_i s_i &= 0 \quad (1 \leq i \leq n) \end{aligned}$$

are in fact the *KKT*-conditions for *each* of the problems – the primal as well as the dual.

ii) Show that the dual problem that you associate to the dual problem above is again the primal problem.

Problem 3. Let $\mathbf{x} \in \mathbb{R}^n$ be admissible for the (primal) LP

$$\min\ (\mathbf{c}, \mathbf{x}) \qquad \text{subject to} \qquad \boldsymbol{A}\mathbf{x} = \mathbf{b} \quad \text{and} \quad \mathbf{x} \geq \mathbf{0}\,, \tag{P}$$

and $(\boldsymbol{\lambda}, \mathbf{s}) \in \mathbb{R}^m \times \mathbb{R}^n$ be admissible for the dual problem

$$\max\ (\mathbf{b}, \boldsymbol{\lambda}) \qquad \text{subject to} \qquad \boldsymbol{A}^\top\boldsymbol{\lambda} + \mathbf{s} = \mathbf{c}, \quad \mathbf{s} \geq \mathbf{0}\,. \tag{D}$$

Suppose that $(\mathbf{x}, \mathbf{s}) \leq \epsilon$ for some $\epsilon > 0$. Show the following estimates:

i) $(\mathbf{c}, \mathbf{x}^*) \leq (\mathbf{c}, \mathbf{x}) \leq (\mathbf{c}, \mathbf{x}^*) + \epsilon$,

ii) $(\mathbf{b}, \boldsymbol{\lambda}) - \epsilon \leq (\mathbf{b}, \boldsymbol{\lambda}) \leq (\mathbf{b}, \boldsymbol{\lambda}^*)$,

for all primal solutions $\mathbf{x}^* \in \mathbb{R}^n$ and all dual solutions $(\boldsymbol{\lambda}^*, \mathbf{s}^*) \in \mathbb{R}^m \times \mathbb{R}^n$.

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