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Fachbereich Mathematik

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Nichtlineare Optimierung

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Übungsaufgaben 7

Problem 1. Let $f, g : \mathbb{R}^3 \to \mathbb{R}$, via

$$f(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \qquad \text{and} \qquad g(x_1, x_2, x_3) = x_1 + x_2 + x_3 \,.$$

Find a solution of

$$\min_{\mathbf{x} \in \mathscr{X}} f(\mathbf{x}) \,, \qquad \text{where} \qquad \mathscr{X} = \big\{ \mathbf{x} \in \mathbb{R}^3 : \, g(\mathbf{x}) \leq -3 \big\} \,.$$

Problem 2. Let $m \in \mathbb{N}$ be fixed. Consider the problem of finding a sphere in $\mathbb{R}^3$ of minimum radius that contains all given vectors $\{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m\} \subset \mathbb{R}^3$. Formulate this task as inequality constrained minimization problem and derive the corresponding optimality conditions for $m = 3$.

Problem 3. For all $1 \leq i \leq m$ let $f, g_i \in C^1(\mathbb{R}^n)$ be convex, and $\mathbf{h} \in C(\mathbb{R}^n, \mathbb{R}^p)$ be affine. Let $\mathbf{x}^* \in \mathbb{R}^n$ be a solution of

$$\min_{\mathbf{x} \in \mathscr{X}} f(\mathbf{x}) \,, \quad \text{where} \quad \mathbf{x} \in \mathscr{X} := \big\{ \mathbf{x} \in \mathbb{R}^n : \, \mathbf{h}(\mathbf{x}) = \mathbf{0} \quad \text{and} \quad \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \big\} \,.$$

Define

$$\mathscr{T}_{\texttt{strict}}(\mathbf{x}^*) = \Big\{ \mathbf{d} \in \mathbb{R}^n : \, \big(\nabla h_j(\mathbf{x}^*), \mathbf{d}\big) = 0 \quad (1 \leq j \leq p) \,, \big(\nabla g_i(\mathbf{x}), \mathbf{d}\big) < 0 \quad \big(i \in I(\mathbf{x}^*)\big) \Big\} \,.$$

Show that $\mathscr{T}_{\texttt{strict}}(\mathbf{x}^*) \subset \mathscr{T}_{\mathscr{X}}(\mathbf{x}^*)$.

Abgabe: 05.06.2025.