

Exercise Sheet-1

Stochastic Differential Equations
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1. Conditional Expectation of a Discrete Random Variable

Exercise 1.

Let's consider a random experiment where a six-sided fair die is rolled twice. Define the following random variables:

- Let X be the outcome of the first roll. - Let Y be the outcome of the second roll. - Let Z be the product of the two outcomes, i.e., $Z = X \cdot Y$.

Calculate the conditional expectation $E(X|Z = z)$ for a given value $z = 6$, and prove that it satisfies the properties of an expectation. Specifically, calculate $E(X|Z = 6)$, and demonstrate the following:

1. Compute $E(X|Z = 6)$ using the definition of conditional expectation.

Exercise 2.

Let X be a discrete random variable representing the number of heads obtained when flipping three fair coins. Calculate the conditional expectation $E(X|X \geq 2)$ and prove that it satisfies the properties of an expectation.

2. Conditional Expectation of a Continuous Random Variable

Exercise 1.

Consider a continuous random variable Y with probability density function (PDF) given by:

$$f_Y(y) = \begin{cases} 2(1 - y), & \text{for } 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Let X be another continuous random variable defined as $X = 2Y$. Calculate the conditional expectation $E(X|Y = y)$ and prove that it satisfies the properties of an expectation.

Exercise 2

Consider a continuous random variable X with probability density function (PDF) given by:

$$f_X(x) = \begin{cases} 2x, & \text{for } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Let Y be another continuous random variable defined as $Y = X^2$. Calculate the conditional expectation $E(Y|X = x)$ and prove that it satisfies the properties of an expectation.

Deadline: 2nd Nov 2023, 12:00.