EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistisches Lernen

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Homework 3

Problem 1. Let $Z \sim \mathcal{B}(n,p)$ be binomially distributed with parameters n and p. Then

(i) $\mathbb{E}\big[\frac{1}{1+Z}\big] \leq \frac{1}{(n+1)p}$.

(ii) $\mathbb{E}\big[\frac{1}{Z}\mathbb{1}_{\{Z>0\}}\big] \leq \frac{2}{(n+1)p}$.

Problem 2. Let the random variable $Z \sim \mathcal{B}(n,p)$ be binomially distributed with parameters n and p. Then

$$\mathbb{E}\Big[\frac{1}{Z}\mathbb{1}_{\{Z>0\}}\Big] \geq \frac{1}{np}\big(1-(1-p)^n\big)^2 .$$

Problem 3. Let a rectangle partition consist of rectangles with side lengths $h_{n,1},\dots h_{n,d}$. Prove weak universal consistency of the related partitioning estimator m_n for

$$\lim_{n\uparrow\infty} h_{n,j} = 0 \quad (1 \leq j \leq d) \qquad \text{and} \qquad \lim_{n\uparrow\infty} nh_{n,1}\cdots h_{n,d} = \infty .$$

Problem 4. In the lecture, we verified weak universal consistency of a regression estimator m_n under certain assumptions. Revisit the proof and show that the same property holds *without assumption (i)* in case that the regression function is uniformly continuous and the conditional variance function σ^2 is bounded.

Date of Submission: 12.00 on 04.11.2026.