EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistical Learning 1

Summer semester 2024 **Tübingen, 10.06.2024**

Assignment 8

Problem 1
To obtain weak universal consistency for the k-NN estimator, we verify the assumptions in Stone's theorem. In Assginment 7 Problem 1, we have already shown conditions (ii) and $(iv)_2$ and condition (iii) was proved in the lecture. Show the remaining assumptions under the conditions

$$k_n \uparrow \infty \text{ and } \frac{k_n}{n} \downarrow 0 \text{ for } n \uparrow \infty.$$

Problem 2
Show that for the support of X holds

a) $\mathbb{P}[X \in \mathrm{supp}(\mathbb{P}_X)] = 1$

b) $\mathrm{supp}(\mathbb{P}_X)]$ is closed.

Problem 3
Let $\mathcal{S} \equiv \mathrm{supp}(\mathbb{P}_X)$. The following estimate is to prove rates for the kernel estimator. Show that a constant $C_d \equiv C(d, \mathrm{diam}(\mathcal{S})) > 0$ exists, such that

$$\int_S \frac{1}{n\mathbb{P}_X[B(x,h_n)]}\mu[dx] \leq \frac{C_d}{n \cdot h_n^d}.$$

Problem 4
For the proof of rates for the kernel estimator, a variance-bias decomposition is used again. Let m_n be a local averaging estimator and m the regression function. Show that

$$\begin{aligned}&\mathbb{E}\big[|m_n(x) - m(x)|^2 | X_1, ..., X_n\big] \\ =&\mathbb{E}\Big[|m_n(x) - \mathbb{E}[m_n(x)|X_1, ...X_n]|^2 \Big| X_1, ..., X_n\Big] + \Big|\mathbb{E}\big[m_n(x)|X_1, ..., X_n\big] - m(x)\Big|^2\end{aligned}$$

holds for every $x \in R^d$.

Date of Submission: 17.06.2024 in the mailbox at 12 noon.