EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistical Learning 1

Summer semester 2024 **Tübingen, 27.05.2024**

Assignment 6

Problem 1
Assume that the weights $\{\alpha_{n,i}\}$ are nonnegative and that the corresponding local averaging estimate is weakly universally consistent. Prove that assumption (iii) in Stone's Theorem is satisfied, i.e.,

$$\lim_{n\to\infty} \mathbb{E}\big[\sum_{i=1}^{n} |\alpha_{n,i}(X)| \mathbb{1}_{\{\|X_i - X\| > a\}}\big] = 0 \;\forall a > 0.$$

Hint: For any fixed $x \in \mathbb{R}^d$ and $a > 0$ let f be a non-negative continuous function which is 0 on $B(x, \frac{a}{3})$ and is 1 on $B(x, \frac{2a}{3})^c$. Now choose $Y = f(X) = m(X)$, then

$$\mathbb{1}_{\{X \in B(x, \frac{a}{3})\}} \sum_{i=1}^{n} \alpha_{n,i}(X) f(X_i) \geq \mathbb{1}_{\{X \in B(x, \frac{a}{3}\}} \sum_{i=1}^{n} \alpha_{n,i}(X) \mathbb{1}_{\{\|X_i - X\| > a\}} \xrightarrow{n\to\infty} 0$$

in probability, therefore, for any compact set K,

$$\mathbb{1}_{\{X \in K\}} \sum_{i=1}^{n} \alpha_{n,i}(X) \mathbb{1}_{\{\|X_i - X\| > a\}} \xrightarrow{n\to\infty} 0$$

in probability.

Problem 2
In the lecture, we verify weak universal consistency of the kernel estimator for the naive kernel $K(x) = \mathbb{1}_{\{\|x\| \leq 1\}}$ by validating the four assumptions in the theorem of Stone.
Verify assumptions (ii) and (iii) in Stone's theorem for the kernel estimator in the case of the more general boxed kernels.

Date of Submission: 3.06.2024 in the mailbox at 12 noon.