EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistical Learning 1

Summer semester 2024 **Tübingen, 13.05.2024**

Assignment 5

Problem 1
In the lecture and for the proof of Stone's theorem, we decomposed the weak consistency analysis into three parts, where the last addresses

$$\mathtt{III}^n := \big(\sum_{i=1}^{n} \alpha_{n,i}(\mathbf{X}) - 1\big) m(\mathbf{X}).$$

Use the assumptions of this theorem to show that $\lim_{n\uparrow\infty} \mathbb{E}\big[|\mathtt{III}^n|^2\big] = 0$.

Problem 2
In the lecture, we showed Stone's theorem, which settles weak consistency of local averaging regression estimators m_n. At some stage of it, we needed to establish convergence to zero (for $n \uparrow \infty$) of

$$\sum_{i=1}^{n} \mathbb{E}\Big[|\alpha_{n,i}(\mathbf{X})|^2 \big(Y_i - m(\mathbf{X}_i)\big)^2\Big]$$

The argumentation in the lecture was fast. Please detail the proof.

Problem 3
Let a rectangle partition consist of rectangles with side lengths $h_{n,1}, ..., h_{n,1}$. Prove weak universal consistency of the related partitioning estimator m_n for

$$\lim_{n\to\infty} h_{n,j} = 0 \quad (1 \leq j \leq d) \quad \text{and} \quad \lim_{n\to\infty} n h_{n,1} \cdots h_{n,d} = \infty$$

Date of Submission: 27.05.2024 in the mailbox at 12 noon.