EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

Prof. Dr. Andreas Prohl
Armin Beck

Statistical Learning 1

Summer semester 2024 **Tübingen, 06.05.2024**

Assignment 4

Problem 1
In the lecture we verified weak universal consistency of a regression estimator m_n under certain assumptions. Revisit the proof and show that the same property holds without assumption

(i) There is a constant c such that for every nonnegative measurable function $f : \mathbb{R}^d \to \mathbb{R}$ satisfying $\mathbb{E}[f(\mathbf{X})] < \infty$ and any $n \in \mathbb{N}$,

$$E\Big[\sum_{i=1}^{n} |\alpha_{n,i}(\mathbf{X})| f(\mathbf{X}_i)\Big] \leq c\mathbb{E}[f(\mathbf{X})],$$

in case that the regression function is uniformly continuous and the conditional variance function $\sigma^2(x) = E[(\mathbf{Y} - m(\mathbf{X}))^2 | \mathbf{X} = x]$ is bounded.

Problem 2
In the lecture we defined the emprical measure μ_n for an i.i.d. sample set $(X_1, ..., X_n)$. Let $\mathcal{P}_n$ be a partition of $\mathbb{R}^d$; we call a cell $\mathcal{A}_{n,j} \in \mathcal{P}_n$ to be empty, if $\mu_n[\mathcal{A}_{n,j}] = 0$. Let now M_n be the number of nonempty cells of $\mathcal{P}_n$. Prove that a.s.

$$\frac{1}{n} M_n \xrightarrow{n\to\infty} 0$$

provided that $\lim_{n\to\infty} \frac{|\{j\in\mathbb{N}: \mathcal{A}_{n,j} \cap \mathcal{B}(0,R) \neq \emptyset\}|}{n} = 0$ for each $R > 0$, where $\mathcal{B}(0, R)$ is the ball of radius R around 0.

Date of Submission: 13.05.2024 in the mailbox at 12 noon.