EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistical Learning 1

Summer semester 2024 **Tübingen, 9.07.2024**

Assignment 12

Problem 1
The first proof on strong consistency of the partitioning estimator required boundedness of Y in the tuple (X,Y); the proof without it for the implementable modification $m_n^{'}(x)$ of this estimator requires an exponential estimate for binomial random variables $B \sim B(n,p)$ by Chernov given below — as $n\mu_n[A_n] \sim B(n, \mu[A_n])$ for $A_n \in \mathcal{P}_n$.
Show the following two assertions for $B \sim B(n,p)$.

a) Let $1 > \epsilon > p > 0$. Then

$$\mathbb{P}[B > n\epsilon] \leq \exp\Big(-n\big(\epsilon \log\big(\frac{\epsilon}{p}\big) + (1-\epsilon)\log\big(\frac{1-\epsilon}{1-p}\big)\big)\Big) \leq \exp\Big(-n\big(p - \epsilon + \epsilon\log(\frac{\epsilon}{p})\big)\Big).$$

b) Let $1 > p > \epsilon > 0$. Then

$$\mathbb{P}[B < n\epsilon] \leq \exp\Big(-n\big(\epsilon \log\big(\frac{\epsilon}{p}\big) + (1-\epsilon)\log\big(\frac{1-\epsilon}{1-p}\big)\big)\Big) \leq \exp\Big(-n\big(p - \epsilon + \epsilon\log(\frac{\epsilon}{p})\big)\Big).$$

Date of Submission: 15.07.2024 in the mailbox at 12 noon.