EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Fachbereich Mathematik

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Statistical Learning 2

Summer-Semester 2023 **Tübingen, 24.05.2023**

Homework 4

Problem 1. Let $\mathcal{A} \subset \mathbb{R}^d$ be finite. Show that

(i) $S(\mathcal{A}, n) \leq |\mathcal{A}| \quad \forall\, n \in \mathbb{N}$, and

(ii) $V_{\mathcal{A}} \leq \log_2 |\mathcal{A}|$.

Problem 2. Consider the class of 'two-sided' intervals

$$\mathcal{A} = \big\{(a, b];\, a, b \in \mathbb{R}\big\} \subset \mathcal{B}(\mathbb{R})\,.$$

In the lecture, we showed that $\mathcal{A}$ shatters $\mathcal{G}$, if $|\mathcal{G}| = 2$, but not if $|\mathcal{G}| > 2$. Consequently, its VC dimension is $V_{\mathcal{A}} = 2$. Show that the n-th shatter coefficient of $\mathcal{A}$ is

$$S(\mathcal{A}, n) = 1 + \frac{n(n+1)}{2} \qquad (n \in \mathbb{N})\,.$$

Problem 3. Let $\mathcal{A} \subset \mathcal{B}(\mathbb{R}^2)$ constitute the class of axis-aligned rectangles. How large is its VC dimension $V_{\mathcal{A}}$?

Hint: Recall that to verify that $V_{\mathcal{A}} = n$ requires to show that there exists at least one set of points $\boldsymbol{z}_n \subset \mathbb{R}^2$ that can be shattered by $\mathcal{A}$, but that no set $\boldsymbol{z}_{n+1} \subset \mathbb{R}^2$ of $n+1$ points can be shattered by $\mathcal{A}$.

Date of Submission: 12.00 on 7.06.2023.