

4. Exercise Sheet to Numerical Methods for Quantum Dynamics

The original work related to this sheet is the paper *Computing quantum dynamics in the semiclassical regime* of Christian Lubich and Caroline Lasser (which is found on the webpage).

Exercise 6: Consider a gaussian pulse of the form

$$u(x) = \exp\left(\frac{i}{\varepsilon} \left(\frac{1}{2}(x-q)^T C(x-q) - p^T(x-q) + \zeta\right)\right),$$

where $q \in \mathbb{R}^d, p \in \mathbb{R}^d, C = C^T \in \mathbb{C}^{d \times d}$, $\text{Im } C$ is positive definite, $\zeta \in \mathbb{C}$. Show the following two identities

$$\langle u | xu \rangle = q, \quad \langle u | -i\varepsilon \nabla_x u \rangle = p.$$

Exercise 7: Prepare a short blackboard presentation, which presents Lemma 3.7 (on page 17 in the paper from above).

Exercise 8: Prepare a short blackboard presentation, which presents Lemma 3.12 (on page 21 in the paper from above).

Exercise 9: Prepare a short blackboard presentation, which presents Lemma 3.13 (on page 22 in the paper from above).