

2. Exercise Sheet to Numerical Methods for Quantum Dynamics

Exercise 3: Consider the Dirac lowering operator $A = \frac{1}{\sqrt{2}}(q + ip)$.

- (a) Show that $\phi_0 = e^{-x^2/2}$ is in the kernel of A .
- (b) Show that ϕ_0 spans the kernel of A .

Hint: Assume $\phi \in \ker A$. Write $\phi(x) = \rho(x)\phi_0(x)$ and show that $\partial_x \rho = 0$.

The next two exercises are to be understood as a “mini-seminar” and are concerned with an approximation result about the Galerkin error.

Exercise 4: Prepare a blackboard talk, which presents

- The contents of **Theorem III.1.3 (Galerkin Error)** in the book and,
- part (a) of the proof.

Exercise 5: Prepare a blackboard talk, which presents the part (b) of the proof of **Theorem III.1.3 (Galerkin Error)** in the book.