Universität Tübingen
Mathematisches Institut
Prof. Dr. Christian Lubich

Tübingen, July 13th, 2018

12th Exercise sheet – Numerics for instationary differential equations

Exercise 32: A complex Gauss function is given by

$$\psi_0(x) = \exp(-a|x-\bar{q}|^2 + \mathrm{i}\bar{p}(x-\bar{q}) + c),$$

where $x \in \mathbb{R}$, $\bar{p}, \bar{q} \in \mathbb{R}^n$, $a \in \mathbb{C}$ with $\Re(a) > 0$ and $c \in \mathbb{C}$, such that $\|\psi_0\|_{L^2} = 1$

Prove: If the initial data of the Schrödinger equation $\mathrm{i}\partial\psi/\partial t = -\frac{1}{2m}\Delta\psi$ is a complex Gauss function, then the solution remains a complex Gauss function for all t with $\bar{p}(t) = \bar{p}(0), \bar{q}(t) = \bar{q}(0) + t\bar{p}(t)/m$

Hint: Make an ansatz for $\bar{p}(t)$ and $\bar{q}(t)$ as given above and determine a and c from the ordinary differential equations you get from the Schrödinger equation.

Exercise 33: In the lecture you have shown the following a priori estimate for the Strang splitting: Let $\psi(t)$ be the solution of the Schrödinger equation with Hamiltonian $H = T + V$ and initial data ψ_0 and let $V(x)$ be sufficiently regular. Then there exists a $\tau_0 > 0$ and a $c > 0$, such that for all $\tau < \tau_0$ with $t_n = n\tau$, the n-th approximation ψ_n of the Strang splitting satisfies

$$\|\psi_n - \psi(t_n)\|_{L^2} \le c t_n \tau^2 \max_{0 \le t \le t_n} \|\psi(t_n)\|_{L^2}.$$

Formulate and proof an a priori estimate for the Lie Trotter splitting.

Solutions are discussed on July 20th.
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