Universität Tübingen
Mathematisches Institut
Prof. Dr. Christian Lubich

Tübingen, May 4th, 2018

3rd Exercise sheet – Numerics for instationary differential equations

Exercise 7:

Consider the differential equation

$$y' = Ay + g(t, y),$$

where

- $\langle Av, v\rangle \le \mu \|v\|^2$, for all $v \in \mathbb{R}^d$,
- and g satisfies a Lipschitz condition with constant L.

We apply a *linearly implicit Euler-method*

$$y_{n+1} = y_n + h\left(Ay_{n+1} + g(t_n, y_n)\right)$$

Prove: If $\mu + L \le 0$, then both the differential equation and the method are contractive.

Exercise 8:

(a) Prove: Every contractive Runge–Kutta method is A-stable.

(b) Is the following implicit Runge–Kutta method contractive?

$$\begin{array}{c|cc} 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline & \frac{1}{4} & \frac{3}{4} \end{array}$$

Exercise 9:

Show that s-step Gauss and Radau collocation methods satisfy the coercivity condition

$$\exists\, D = \mathrm{diag}(d_i) > 0 : \exists\, \alpha > 0 : \langle u, A^{-1}u\rangle_D \ge \alpha \langle u, u\rangle_D, \quad \forall u \in \mathbb{R}^s.$$

Hint: Consider the conditions

(1) $\sum_{i=1}^{s} b_i c_i^{k-1} = \frac{1}{k}, \quad 1 \le k \le p,$

(2) $\sum_{j=1} a_{ij} c_j^{k-1} = \frac{c_i^k}{k}, \quad 1 \le i \le s,\ 1 \le k \le q,$

for convenient choices of p and q and

$$D = B(C^{-1} - I_s) \qquad \text{resp.} \qquad D = BC^{-1},$$

where $B = \mathrm{diag}(b_i)$ and $C = \mathrm{diag}(c_i)$.

Solutions are discussed on May 11th.
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