Universität Tübingen
Mathematisches Institut
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Tübingen, November 4, 2025

3. Exercise Sheet for Numerics of Stationary Differential Equations

Exercise 7:

Suppose there is given a 3-point boundary value problem of the form

$$y' = f(y), \qquad r\big(y(a), y(\tau), y(b)\big) = 0, \qquad a < \tau < b.$$

Let $y^\star$ be a solution to the problem. Give a sufficient condition such that $y^\star$ is locally unique.

Hint: Follow the ideas from §2 from the lecture.

Exercise 8:

Formulate the multi–shooting method for the 3-point boundary value problem from Exercise 7. How does the linear system of equation, that is to be solved in each Newton-step, look like?

Hint: Suppose that τ is a partition point.

Programming Exercise 2 :

Implement the collocation method from Exercise 6 ($s = 1$, $c_1 = \frac{1}{2}$) as a multi–shooting method as in Exercise 7 for the boundary value problem

$$y'' + t^{-1}y' - 4t^{-2}y = 0, \qquad y(1) = 2, \quad y(2) = 17/4.$$

Divide the interval $[1, 2]$ equidistant in m sub-intervals $[t_j, t_{j+1}]$, $0 \le j \le m - 1$. Test your program for $m = 2^i$, $1 \le i \le 6$ and determine $\max_{0\le j\le m} |u(t_j) - y(t_j)|$, where u is the approximation from the collocation method to the exact solution y. Which convergence rate can you observe? Appropriate initial values for $y(t_j)$ — without any knowledge of the exact solution — can be obtained via linear interpolation (and for $y'(t_j)$ choose simply 0).

Hint: The exact solution of the boundary value problem is $y(t) = t^2 + t^{-2}$.

Solutions are discussed on Tuesday November 11, 2025
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.