

For the local load vector:

$$\begin{aligned}\int_E f \varphi_i dx &= \int_{\hat{E}} \hat{f} \circ F^{-1} \hat{\varphi}_i \circ F^{-1} d\hat{x} \\ &= \int_{\hat{E}} \hat{f} \hat{\varphi}_i |\det B| d\hat{x} \\ \text{approx. by quadrature} &\approx \frac{1}{2} \hat{f}\left(\frac{1}{3}, \frac{1}{3}\right) \hat{\varphi}_i\left(\frac{1}{3}, \frac{1}{3}\right) |\det B|\end{aligned}$$

For the local stiffness matrix:

Let

$$B^{-1} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

Then:

$$\begin{aligned}\int_E \nabla \phi_i \nabla \phi_j dx &= \dots \\ &= |\det B| [(C_{11}^2 + C_{12}^2)A^{xx} + (C_{21}^2 + C_{22}^2)A^{yy} \\ &\quad + (C_{11}C_{21} + C_{12}C_{22})(A^{xy} + A^{yx})]\end{aligned}$$

where

$$A^{ij} = \int_{E_0} \partial_i \varphi \partial_j \varphi dx$$

from exercise 23.