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8. Exercise sheet for numerics of stationary differential equations

Exercise 20:

Assume a triangulation of a bounded domain $\Omega \subset \mathbb{R}^2$. Let u be a function, which is C^1 on each triangle.

Show:

$$u \in H^1(\Omega) \Longleftrightarrow u \in C(\bar{\Omega})$$

Hint: $u \in H^1(\Omega) \Longleftrightarrow u \in L^2(\Omega)$ and u admits weak derivatives (see ex. 18).

Exercise 21:

(a) Give a continuous function on [0,1], which is not an element in $H^1(0,1)$.

(b) Let Ω a ball in $\mathbb{R}^3$ with center in the origin. Show: For $\alpha < 1/2$, the function $u(x) = \|x\|^{-\alpha}$ is in $H^1(\Omega)$.

Exercise 22:

Let $\Omega = [a,b]$ a real interval. Show: $H^1(a,b) \subset C[a,b]$.
Hint:

(a) Show: $|v(x)| \leq C\|v\|_1$ for $v \in C^\infty[a,b]$.

(b) Use the density of C^∞ in H^1 with respect to the $\|\cdot\|_1$-norm.

Discussion of the sheet on 11.12.2023.