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3. Exercise sheet for numerics of stationary differential equations

Exercise 5:

Show that for the solution of the multi shooting equation it holds:

$$\Delta x_j = \sum_{l=0}^{m-1} G_{jl} F_l - E_{m-j}^{-1} F_m,$$

with

$$E_{m-j} := AR_0^{-1} \cdots R_{j-1}^{-1} + BR_{m-1} \cdots R_j$$

and

$$G_{jl} = \begin{cases} E_{m-j}^{-1} AR_0^{-1} \cdots R_l^{-1} & l < j, \\ -E_{m-j}^{-1} BR_{m-1} \cdots R_{l+1} & l \geq j, \end{cases}$$

where we define the empty product as I. The matrix (G_{jl}) can then be seen as a discrete analogous of the Green's function from exercise 2.

Hint: Show and use $E_{m-(j+1)} R_j = E_{m-j}$.

Exercise 6:

How does a Runge-Kutta method, which is equivalent to a collocation method with a (singel) node $c_1 = 1/2$, look like?

Exercise 7:

Consider the linear boundary value problem:

$$y' = C(t)y, \qquad Ay(a) + By(b) = r.$$

If you apply the collocation method from exercise 6 as a multiple shooting method to the boundary value problem, you obtain a linear equation system. Give the linear equation system.

Discussion of the sheet on 06.11.2023.