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12. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 34: Understand a simplex step using the following linear optimization task as an example

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 12 \\ x &\geq 0 \\ x_1 + x_2 + x_3 &= \min! \end{cases}$$

$x = (12, 0, 0)^T$ is chosen as the initial edge. Consider the following:

(a) How does the cost $x_1 + x_2 + x_3$ change as the second component of x grows?

(b) How does the cost $x_1 + x_2 + x_3$ change as the third component of x grows?

(c) Which component of x is increased in the first step of the simplex procedure and why?

(d) What does the new edge look like then?

(e) Why is this edge optimal?

Exercise 35: Determine a corner for the linear optimization task

$$\begin{cases} x_1 - x_2 - x_3 &= 6 \\ x &\geq 0 \\ x_1 + x_2 + x_3 &= \min! \end{cases}$$

with the "phase I" of the simplex algorithm.

Exercise 36: (2 Points) Consider for $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$ with $n \geq m$ the problem

$$\begin{cases} Ax = b, \ x \geq 0 \\ c^T x = \min! \end{cases}$$

with admissible set $D = \{y \in \mathbb{R}^n; \ Ay = b, y \geq 0\}$. Show: if $\operatorname{rank}(A) = m$, then there always exists a corner $x \in D$.
Hint:

a) $x \in D$ is a corner if and only if x cannot be represented as a convex combination of two points $y, z \in D$, i.e. if

$$x = (1-\lambda)y + \lambda z \text{ with } \lambda \in [0,1], \ y, z \in D \quad \Rightarrow \quad x = y = z.$$

b) Consider a given $x \in \mathbb{R}^n$ with $x_i \geq 0$ for $i \in \mathcal{I}$ and $x_i = 0$ for $i \notin \mathcal{I}$, where $\mathcal{I} \subseteq \{1, \ldots, n\}$ is an m-elementary subset, and $A_{\mathcal{I}} = (Ae_j)_{j \in \mathcal{I}}$ is a corner if $A_{\mathcal{I}}$ is regular. Then consider a vector $x \in D$ with minimal number of non-vanishing components and show that the reduced matrix $A_{\mathcal{I}}$ has full rank, where $\mathcal{I}$ are those indices whose associated x-component does not vanish. Finally, use $\operatorname{rank}(A) = m$.

Solutions are discussed on Tuesday 15.07.2025.
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.