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6. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 16: A square matrix A is called stochastic if all of its entries are non-negative, and the entries of each column sum to 1. Prove, that the largest eigenvalue of a stochastic matrix is 1.

Exercise 17: How can one calculate the eigenvectors of an upper triangular matrix with pairwise different diagonal elements? Give an algorithm in pseudo code. Further, sketch how to calculate the eigenvectors of a diagonalizable matrix A with pairwise different eigenvalues.

Exercise 18: Show: The QR-decomposition of a complex square matrix is unique up to a multiplication with a diagonal matrix, i.e.

$$QR = (QD)(D^{-1}R),$$

where $D = \operatorname{diag}(d_1, \dots, d_n)$, $|d_i| = 1$ for all $i = 1, \dots, n$.
Note: Let $A = QR$, if A is a real square matrix then Q is orthonormal, whereas if A is complex then it is just unitary.

Exercise 19:

a) Transform the matrix

$$\begin{pmatrix} 2 & 7 & 3 \\ 3 & 4 & 1 \\ 4 & 2 & -2 \end{pmatrix}$$

through Householder-transformations to a Hessenberg matrix.

b) Let

$$A = \begin{pmatrix} 12 & -2 & 9 \\ -6 & 0 & -3 \\ 7 & -7 & 8 \end{pmatrix} \quad \text{and} \quad Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & * & * \\ 1 & * & * \\ 0 & * & * \end{pmatrix} \in \mathbb{R}^3$$

with $Q^T Q = I$ and $Q^T A Q = H$ in Hessenberg form. Compute H and Q.

Solutions are discussed on Tuesday 27.05.2025.
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.