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5. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 14: If B is normal and A is any $n \times n$ matrix, show that for every eigenvalue λ of A there is an eigenvalue μ of B with

$$|\lambda - \mu| \leq \|A - B\|_2$$

Hint: Show first that if λ is not an eigenvalue of B, then

$$\|(\lambda I - B)^{-1}\|_2 = \frac{1}{\min\limits_{\mu \in \lambda(B)} |\lambda - \mu|}$$

Exercise 15: Calculate the eigenvalues of the $n \times n$ matrix $\widetilde{A} = A + \epsilon C$ with

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \dots & \vdots & 0 & 1 \\ 0 & \dots & \dots & 0 & 0 \end{pmatrix}, \qquad C = \hat{e}_n \hat{e}_1^T$$

What do you get for $n = 8$ and $\varepsilon = 10^{-8}$?

Programming Exercise 4: Code the direct power iteration (=power method). Plot the errors for the following test matrices:

```
n = length(d);
S = triu(diag(n:-1:1,0) + ones(n,n));
A = S*diag(d,0)*inv(S);
```

and

```
n = length(d);
z = diag(sqrt(1:n),0) + ones(n,n);
[Q, R] = qr(z);
B = Q*diag(d,0)*Q';
```

with

1. `d = (1:10)';`
2. `d = [ones(9,1); 2];`
3. `d = 1-2.*(-(1:0.5:5));`

Programming Exercise 5: Code the inverse iteration (=inverse power method). Plot the errors for the test matrices from programming exercise 4.

Solutions are discussed on Tuesday 20.05.2025.
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.