2. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 5: (Sorting Problem)

A file of $N = 2^L$ names is to be sorted alphabetically. Propose an algorithm that does this in $\mathcal{O}(N \log N)$ operations. Assume that names can be compared in $\mathcal{O}(1)$ operations. Hint: Divide et impera!

Exercise 6:

- (a) Let $x = (x_0, x_1, \dots, x_{N-1}) \in \mathbb{R}^N$ (thus x_j real). Show that: $\hat{x}_{-k} = \overline{\hat{x}_k}$ for $k \in \mathbb{Z}$, where \hat{x} is the discrete Fourier Transform of x.
- (b) If $x \in \mathbb{C}^N$ is an even sequence (i.e., $x_{-k} = x_k$ for all $k \in \mathbb{Z}$), then its transformation \hat{x} is even too. If x is odd (d.h., $x_{-k} = -x_k$ for all $k \in \mathbb{Z}$), then its transformation \hat{x} is odd too.

Exercise 7: (Chebyshev-Polynomials)

Let $t_0 \in \mathbb{R}$ with $|t_0| > 1$. Show that: Among all polynomials p_k of degree k satisfying $p_k(t_0) = 1$ (normalized at t_0), the quantity

$$\max_{t\in[-1,1]}|p_k(t)|$$

is minimized by the Chebyshev polynomial normalized at t_0 , namely $T_k(t)/T_k(t_0)$.

Programming Exercise 1: Implement the fast Fourier transform (FFT) (without using premade functions for packages). You may assume that the length of the input vector is a power of two. <u>Note</u>: Implement the FFT algorithm recursively (i.e. the function should call itself).

Solutions are discussed on Tuesday 29.04.2025.

Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.