Universität Tübingen
Mathematisches Institut
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1. Exercise Sheet for Algorithms in Numerical Mathematics

Exercise 1: (Fourier Series)

Let $c = (c_n)_{n=-\infty}^{\infty}$ be n absolutely summable sequence of complex numbers. Show

(a) that the inversion formula holds

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} \hat{c}(\theta)e^{-in\theta}\,\mathrm{d}\theta, \qquad \text{where} \qquad \hat{c}(\theta) = \sum_{n=-\infty}^{\infty} c_n e^{in\theta},$$

(b) that Parseval's equation holds

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{2\pi}\int_0^{2\pi} |\hat{c}(\theta)|^2\,\mathrm{d}\theta$$

Exercise 2: Prove the convolution theorem:
Let $c = (c_n)_{n\in\mathbb{Z}}$ and $d = (d_n)_{n\in\mathbb{Z}}$ be absolutely summable sequences. Then, the convolution is also absolutely summable and the following holds:

$$\widehat{(c * d)}(t) = \hat{c}(t)\hat{d}(t) \quad \text{for all } t \in \mathbb{R}.$$

Exercise 3: (Sinus-/Cosine series)

Prove that the Fourier series of a continuous $2\pi-$periodic function $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{int}$ admits an equivalent representaiton of the form

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nt) + b_n \sin(nt)\right)$$

Write a_n, b_n as a function of c_n. How can one calculate a_n and b_n from $f(t)$? What happens for even $(f(t) = f(-t))$ and odd $(f(t) = f(-t))$ functions?

Exercise 4: (Cesàro-Sums)

We define the *Cesàro-sum* for a sequence $(a_n)_{n\in\mathbb{N}}$ via

$$s_n = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

Prove that, the convergence of the sequence $(a_n)_{n\in\mathbb{N}}$ to some value a implies convergence of the sequence $(s_n)_{n\in\mathbb{N}}$ to a, but convergence of $(s_n)_{n\in\mathbb{N}}$ does not imply convergence of $(a_n)_{n\in\mathbb{N}}$.

Solutions are discussed on Tuesday 22.04.2025.
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.