Universität Tübingen
Mathematisches Institut
Prof. Dr. Christian Lubich

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6. Exercise sheet for Algorithmen der Numerischen Mathematik

Exercise 17: (Computations with Hessenberg)

a) Transform the matrix

$$\begin{pmatrix} 2 & 7 & 3 \\ 3 & 4 & 1 \\ 4 & 2 & -2 \end{pmatrix}$$

through Householder-transformations to a Hessenberg matrix.

b) Let

$$A = \begin{pmatrix} 12 & -2 & 9 \\ -6 & 0 & -3 \\ 7 & -7 & 8 \end{pmatrix} \quad \text{and} \quad Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & * & * \\ 1 & * & * \\ 0 & * & * \end{pmatrix} \in \mathbb{R}^3$$

with $Q^T Q = I$ and $Q^T A Q = H$ in Hessenberg form. Compute H and Q.

Exercise 18:

Give an algorithm, which computes the QR-decomposition of a symmetric, tridiagonal matrix of dimension n in $O(n)$.

Exercise 19: (Francis QR-step)

In the algorithm for the computation of complex eigenvalues of real matrices, presented in the lecture, one uses the first column of the matrix M_k.

a) Give an algorithm that computes $M_k e_1$ in as few operations as possible.

b) Then give an algorithm that computes the reflection $Q(M_k e_1) = \alpha e_1$ with a Householder matrix Q as efficiently as possible.

Programming exercise 5:

Implement the algorithm of exercise 18 and test it for at least one matrix.

Solutions are discussed on Wednesday 07.06.2023.
Contact person: Dominik Sulz - when you have questions just come to my office (C3P16) or write me an email.