

## 5. Exercise sheet for Algorithmen der Numerischen Mathematik

### Exercise 15: (Calculation of eigenvectors)

How can one calculate the eigenvectors of an upper triangular matrix with pairwise different diagonal elements? Give an algorithm in pseudo code. Further, sketch how to calculate the eigenvectors of a diagonalizable matrix  $A$  with pairwise different eigenvalues.

### Exercise 16: (Uniqueness of the QR-decomposition)

Show: The  $QR$ -decomposition is unique up to a multiplication with a diagonal matrix, i.e.

$$QR = (QD)(D^{-1}R),$$

where  $D = \text{diag}(d_1, \dots, d_n)$ ,  $|d_i| = 1$  for all  $i = 1, \dots, n$ .

### Programming exercise 3:

Code the direct power iteration (=power method). Plot the errors of the following test matrices:

```
n = length(d);  
S = triu(diag(n:-1:1,0) + ones(n,n));  
A = S*diag(d,0)*inv(S);
```

and

```
n = length(d);  
z = diag(sqrt(1:n),0) + ones(n,n);  
[Q R] = qr(z);  
B = Q*diag(d,0)*Q';
```

with

1.  $d = (1:10)'$ ;
2.  $d = [\text{ones}(9,1); 2]$ ;
3.  $d = 1 - 2.^{-(1:0.5:5)}$ ;

### Programming exercise 4:

Code the inverse iteration (=inverse power method). Plot the errors of the test matrices from programming exercise 3.

**Solutions are discussed on Wednesday 24.05.2023.**

Contact person: Dominik Sulz - when you have questions just come to my office (C3P16) or write me an email.