Universität Tübingen
Mathematisches Institut
Prof. Dr. Christian Lubich

Tübingen, 19.04.2023

1. Exercise sheet for Algorithmen der Numerischen Mathematik

Exercise 1: (Sine-/Cosine series)

Proof that the Fourier series of a continuous 2π-periodic function $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{int}$ admits an equivalent representation of the form

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos(nt) + b_n \sin(nt))$$

Write a_n, b_n as a function of c_n. How can one calculate a_n and b_n from $f(t)$? What happens for even and odd functions ($f(t) = f(-t)$ and $f(t) = -f(-t)$)?

Exercise 2: Proof the convolution theorem:
If the complex sequences $c = (c_n)_{n\in\mathbb{Z}}$ and $d = (d_n)_{n\in\mathbb{Z}}$ are absolutely summable, then the convolution $c * d$ is absolutely summable and it holds

$$\widehat{(c * d)}(t) = \hat{c}(t)\hat{d}(t) \quad \text{for all } t \in \mathbb{R}.$$

Exercise 3: (Cesàro-sums)

For a sequence $(a_n)_{n\in\mathbb{N}}$ we define the *Cesàro-sum* via

$$s_n = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

Proof the following: Convergence of the sequence $(a_n)_{n\in\mathbb{N}}$ to an a implies convergence of the sequence $(s_n)_{n\in\mathbb{N}}$ to a, but convergence of $(s_n)_{n\in\mathbb{N}}$ does not imply convergence of $(a_n)_{n\in\mathbb{N}}$.

Exercise 4:

(a) Let $x = (x_0, x_1, \ldots, x_{N-1}) \in \mathbb{R}^N$ (thus x_j *real*). Proof: $\hat{x}_{-k} = \overline{\hat{x}_k}$ for $k \in \mathbb{Z}$, where $\hat{x}$ is the discrete Fourier-transform of x.

(b) If $x \in \mathbb{C}^N$ is a even sequence (i.e., $x_{-k} = x_k$ for all $k \in \mathbb{Z}$) then its Fourier-transform $\hat{x}$ is even. If x is odd (i.e., $x_{-k} = -x_k$ for all $k \in \mathbb{Z}$) then its Fourier-transform $\hat{x}$ is odd.

Solutions are discussed either Wednesday 26.04.2023 or Friday 28.04.2023.
Contact person: Dominik Sulz - when you have questions just come to my office (C3P16) or write me an email.