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9. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

Exercise 21. Let us introduce the L^2 inner product with *lumped masses*:

$$m_{\mathrm{lm}}(b_h, w_h) = \int_{\Gamma_h(t)} \widetilde{I}_h(v_h w_h),$$

where $\widetilde{I}_h$ is the interpolation operator on $\Gamma_h(t)$.

Instead of the usual definition of the mass matrix $\mathbf{M}$, consider the mass matrix with *mass lumping*

$$\bar{\mathbf{M}}|_{jk} = m_{\mathrm{lm}}(\phi_k, \phi_j) = \int_{\Gamma_h(t)} \widetilde{I}_h(\phi_k \phi_j), \qquad \text{with} \quad j,k =, 1 \dots, N.$$

What structure does $\bar{\mathbf{M}}$ has?

Prove that $\bar{\mathbf{M}}|_{kk} = \sum_{j=1}^N \mathbf{M}|_{kj}$.

Exercise 22. Consider the parabolic PDE on a stationary surface Γ:

$$\begin{aligned} \partial_t u - \Delta_\Gamma u &= -\beta u^\alpha, && \text{on } \Gamma \\ u(\cdot, 0) &= u^0, && \text{on } \Gamma \end{aligned}$$

with initial data $u^0 \geq 0$ pointwise, and with constants $\alpha \geq 1$ and $\beta \geq 0$.

Formulate the semi-discrete problem corresponding to the above PDE.

Derive the following matrix-vector formulation of the semi-discrete problem:

$$\bar{\mathbf{M}}\dot{\mathbf{u}}(t) + \mathbf{A}\mathbf{u}(t) = -\beta \bar{\mathbf{M}}(\mathbf{u}(t))^\alpha.$$

Please, bring your laptops along!

Discussed on the tutorials on 10.07.2018.