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## 7. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

**Exercise 16.** An evolving surface $\Gamma(t)$ is given directly by the mapping $X(\cdot, t) : \Gamma^0 \to \mathbb{R}^3$:

$$\Gamma(t) = \left\{ x \in \mathbb{R}^3 \;\middle|\; \text{with} \quad x_1 = p_1 + \max\{0, p_1\}, x_2 = \frac{g(p,t)p_2}{\sqrt{p_2^2 + p_3^2}}, x_3 = \frac{g(p,t)p_3}{\sqrt{p_2^2 + p_3^2}} \;\middle|\; p \in \Gamma^0 \right\},$$

where the initial surface $\Gamma^0$ is the unit sphere, and the function

$$g(p,t) = e^{-2t}\sqrt{p_2^2 + p_3^2} + (1 - e^{-2t})\Big( (1 - p_1^2)(p_1^2 + 0.05) + p_1^2\sqrt{1 - p_1^2} \Big).$$

Write a short code which visualises the surface evolution using the direct mapping. As an initial triangulation use a mesh from a previous programming exercise.

*Hint.* The code is indeed short. Apart from the information given above, nothing else is needed.

**Exercise 17.** Visualise the evolving surface $\Gamma(t)$ from Exercise 12, using the computed velocity, and the pseudo-code from part (c).

Please, download the code `https://na.uni-tuebingen.de/ex/surfPDE_ss18/func_v.m`, and bring your laptop along!

**Exercise 18.** (a) Derive the fully discrete scheme a $k$-step BDF method for the numerical solution of the heat equation on an evolving surface. Recall that the corresponding matrix vector formulation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\mathbf{M}(t)\mathbf{u}(t)\Big) + \mathbf{A}(t)\mathbf{u}(t) = \mathbf{b}(t),$$

with given initial data.

(b) Write a pseudo-code.

**Discussed on the tutorials on 26.06.2018**.