

5. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

Exercise 9. (a) Collect the some of the most important properties of the Radau IIA methods for stage number $s \in \mathbb{N}$.

Hint: Butcher tableau, order, stability function, various stability properties, etc.

(b) Collect the most important properties of the k -step BDF methods ($k \in \mathbb{N}$).

Hint: Coefficients, order, stability regions, various stability properties, etc.

Any sources, with correct references, are allowed to be used.

Exercise 10. Formulate the block-matrix form of a general Runge–Kutta method (with a given Butcher tableau) from the lecture (when it was applied to the heat equation on stationary surfaces). (Collect the internal stages \mathbf{U}^{ni} into a vector, formulate the corresponding linear equation system. Use Kronecker products \otimes .)

Write a simple pseudo-code for the fully discrete numerical solution.

Exercise 11. Reformulate the k -step BDF method (for $k \leq 6$) from the lecture (when it was applied to the heat equation on stationary surfaces) as

$$\mathbf{L}\mathbf{u}^n = \mathbf{q}^n \quad (n \geq k).$$

In particular, determine the matrix \mathbf{L} and the vector \mathbf{q}^n (both can depend on the coefficients of the method, and \mathbf{q}^n may also depend on the past values of \mathbf{u}^j).

Write a simple pseudo-code for the fully discrete numerical solution.