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4. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

Exercise 9. Using surface finite elements, discretise the elliptic problem

$$-\Delta_\Gamma u = f \qquad \text{on} \qquad \Gamma,$$

where $f \in L^2(\Gamma)$ with $\int_\Gamma f = 0$. Consider the discrete surface Γ_h given.
Determine:

(a) the discrete finite element space S_h;

(b) and the linear equation system $Au = b$.

Programming exercise 1. Using surface finite elements discretise the problem

$$-\Delta_\Gamma u + \mu u = f \qquad \text{on} \qquad \Gamma,$$

where $\mu > 0$. The surface Γ and the right-hand side function f are given, as follows:

Γ: As a triangulation Γ_h (generated in DistMesh), through the arrays `Elements`, `Nodes`;

f: As a Matlab function `func_f.m`.

(a) Implement the function `[A,M]=surface_assembly(Elements,Nodes)` computing the stiffness and mass matrix, with the help of the Pseudocode discussed at the Tutorials, and with the help of Exersice A5, A6 und A7. The load vector can be approximated by

$$\boldsymbol{b}|_j = \int_{\Gamma_h} f_h \phi_j \approx \int_{\Gamma_h} \widetilde{I}_h f \phi_j = (\boldsymbol{M}\boldsymbol{f})_j$$

(b) Solve the corresponding linear equation system on the different meshes

$$\begin{matrix} \texttt{Elements\_j.txt} \\ \texttt{Nodes\_j.txt} \end{matrix} \qquad j = 1, 2, ..., M$$

(c) Compute the following errors of the numerical solution, when compared to the exact solution given by `func_sol.m`.
As an output generate a convergence plot* in the L^2 norm and the H^1 semi-norm:

$$\|u_h^\ell - u\|^2_{L^2(\Gamma)} \approx \|\boldsymbol{e}\|^2_{\boldsymbol{M}} = \boldsymbol{e}^T \boldsymbol{M} \boldsymbol{e},$$
$$\|\nabla(u_h^\ell - u)\|^2_{L^2(\Gamma)} \approx \|\boldsymbol{e}\|^2_{\boldsymbol{A}} = \boldsymbol{e}^T \boldsymbol{A} \boldsymbol{e}.$$

* As discussed.

The functions and the grid arrays can be found at `https://na.uni-tuebingen.de/ex/surfPDE_ss18/PA1.zip`.

Discussed on the tutorials on 29.05.2018.
The programming exercise is due on 29.05.2018, 12 s.t.