Universität Tübingen
Mathematisches Institut
Dr. Balázs Kovács

Tübingen, den 08.05.2018

## 3. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

**Exercise 5.** (a) Determine the first order local basis functions on the reference triangle (reference element) $E_0$, given by $(0,0), (1,0)$ and $(0,1)$ as its nodes.

(b) Compute the corresponding local matrices:

$$\int_{E_0} \phi_i \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_y \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_y \phi_j \qquad i, j = 1, 2, 3.$$

*Hint:* Use symbolic calculations in Matlab or Maple/Sage, instead of computing by hand.

**Exercise 6.** Determine the affine linear transformation between the reference element $E_0$ and an arbitrary triangle $E \subset \mathbb{R}^3$. What is the inverse transformation?

*Hint:* Notice that the reference element can be considered to be in $\mathbb{R}^3$ (with $(\cdot, \cdot, 0)$). If needed, the normal vector can be used as well.

**Exercise 7.** With the help of the above affine map from $E$ to $E_0$, transform the following integrals onto $E_0$:

$$\int_E \phi_i \phi_j, \qquad \int_E \nabla_E \phi_i \cdot \nabla_E \phi_j.$$

*Hint:* Use integral transformation.

**Discussed on the tutorials on 15.05.2018**.