

2. Exercise sheet for Numerik für Differentialgleichungen auf Oberflächen

Exercise 2. Install the Matlab package DistMesh (<http://persson.berkeley.edu/distmesh/>).

Exercise 3. Consider the elliptic problem

$$-\Delta_{\Gamma} u + \mu u = f \quad \text{on} \quad \Gamma,$$

with $f \in L^2(\Gamma)$, $\mu > 0$, and a sufficiently smooth surface Γ .

Check that the assumptions of the Lax–Milgram theorem are satisfied on the Hilbert space $V = H^1(\Gamma)$. Hence, proving the existence and uniqueness result from the lecture.

Exercise 4. Consider the elliptic problem

$$-\Delta_{\Gamma} u = f \quad \text{on} \quad \Gamma,$$

with $f \in L^2(\Gamma)$, and a sufficiently smooth surface Γ .

(a) Formulate the corresponding weak problem on the Hilbert space $\dot{H}^1(\Gamma) = \{v \in H^1(\Gamma) \mid \int_{\Gamma} v = 0\}$. Under which conditions on f can a solution exist?

(b) Check that the assumptions of the Lax–Milgram theorem are satisfied on the Hilbert space $\dot{H}^1(\Gamma)$. Hence, proving the existence and uniqueness result from the lecture.