Universität Tübingen
Mathematisches Institut
Prof. Dr. Christian Lubich

Tübingen, May 15, 2026

5. Sheet for Numerics of Instationary Differential Equations

Exercise 13:

(a) Give the local linear basis functions for the reference triangle E_0 (reference element) with nodes $(0,0),(1,0)$ and $(0,1)$.

(b) Give the corresponding local matrices

$$\int_{E_0} \phi_i \phi_j, \qquad \int_{E_0} \partial_x \phi_i \, \partial_x \phi_j, \qquad \int_{E_0} \partial_x \phi_i \, \partial_y \phi_j, \qquad \int_{E_0} \partial_y \phi_i \, \partial_x \phi_j, \qquad \int_{E_0} \partial_y \phi_i \, \partial_y \phi_j,$$

for $i,j = 1,2,3$.

Exercise 14:

Give the affine transformation between an arbitrary triangle element $E \subset \mathbb{R}^2$ (with nodes (x_i, y_i), $i = 1,2,3$) and the reference triangle E_0. How does the inverse transformation look like?

Exercise 15:

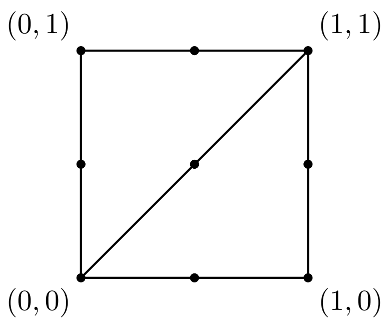
Transform, using the affine map from E to E_0 (see Exercise 24), the following integrals on E to integrals on E_0:

$$\int_E \phi_i \phi_j, \qquad \int_E \nabla \phi_i \cdot \nabla \phi_j.$$

Hint: Use the standard integral transformation (Jacobian) for affine maps.

Exercise 16:

Give the basis functions for a triangle element with a quadratic polynomial space. Give the global basis function corresponding to the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ in the triangulation of the unit square shown below.



Explain how to obtain basis functions for a triangle element with cubic polynomials.

Solutions are discussed on Tuesday May 20, 2026
Tutor: Georgios Vretinaris - if you have question just come to my office (C3P16) or write me an email.