Universität Tübingen
Mathematisches Institut
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## 2st Exercise sheet – Numerics for instationary differential equations

**Exercise 4:** Show that the stability function of a Runge-Kutta method satisfies

$$R(z) = \frac{\det\big(I + z(\mathbb{1}b^T - A)\big)}{\det(I - zA)}.$$

*Hint:* Use (but don't prove) $\det(I + wv^T) = 1 + v^T w$.

**Exercise 5:**

Compute the stability function $R(z)$ of the following Runge-Kutta method (Lobatto IIIC):

$$\begin{array}{c|ccc}
0 & \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \\
\frac{1}{2} & \frac{1}{6} & \frac{5}{12} & -\frac{1}{12} \\
1 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \\
\hline
 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6}
\end{array}$$

Show that this method is $A$-stable.

**Exercise 6:** Consider a collocation method with symmetrically distributed nodes: $c_i = 1 - c_{s+1-i}$ for $i = 1, \ldots, s$. Prove that the stability function of this method satisfied

$$R(z) \cdot R(-z) = 1 \qquad \text{for all } z \in \mathbb{C} \quad \text{(except of the poles)}\ .$$

In particular, $|R(z)| \equiv 1$ on the imaginary axis $\mathrm{i}\mathbb{R}$.
Hints: Let $A$ and $b$ be the coefficient matrix and the coefficient vector of the Runge–Kutta method, respectively, and $\mathbb{1} = (1, \ldots, 1)^T$. Show and use

$$b = Pb, \qquad\qquad A = \mathbb{1}b^T - PAP,$$

where

$$P = \begin{pmatrix} 0 & \ldots & & 0 & 1 \\ 0 & & & \iddots & 0 \\ & & 1 & & \\ 0 & \iddots & & & 0 \\ 1 & 0 & \ldots & & 0 \end{pmatrix}.$$

Then, use exercise 4.

**Solutions are discussed on May 4th.**
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