Universität Tübingen
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## 9. Exercise sheet for numerics of stationary differential equations

**Exercise 23:**

(a) Give the local linear basis functions for the reference triangle $E_0$ (reference element) with nodes $(0,0),(1,0)$ and $(0,1)$.

(b) Give the corresponding local matrices:

$$\int_{E_0} \phi_i \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_x \phi_i \partial_y \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_x \phi_j, \quad \int_{E_0} \partial_y \phi_i \partial_y \phi_j \qquad i,j = 1,2,3.$$

**Exercise 24:**

Give the affine transformation between an arbitrary triangle element $E \subset \mathbb{R}^2$ (with nodes $(x_i, y_i)$ $i = 1,2,3$) and the reference triangle $E_0$. How does the inverse transformation look like?

Geben Sie die affine Transformation zwischen einem beliebigen Dreieckselement $E \subset \mathbb{R}^2$ (mit Knoten $(x_i, y_i)$ $i = 1,2,3$) und dem Referenzdreieck $E_0$ an. Wie sieht die inverse Transformation aus?

**Exercise 25:**

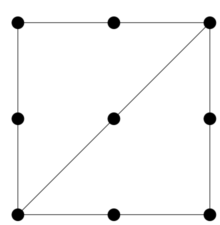
Transform with the help of the affine map from $E$ to $E_0$ (see exercise 24) the following integrals to $E_0$:

$$\int_E \phi_i \phi_j, \qquad \int_E \nabla \phi_i \cdot \nabla \phi_j.$$

Hint: Integral transformation.

**Exercise 26:**

Give the basis functions for a triangle element with a quadratic polynomial space. Give the global basis function corresponding to the point $(\frac{1}{2}, \frac{1}{2})$ in the triangulation of the unit square from below.



Explain how to get the basis functions for a triangle element with cubic polynomials.

**Discussion of the sheet on 18.12.2023**.