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Tübingen, the 20.11.2023

6. Exercise sheet for numerics of stationary differential equations

Exercise 14:

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain. How do you need to modify the finite differences method for the Poisson equation

$$-\Delta u = f \quad \text{in } \Omega\,, \qquad u = g \quad \text{on } \Gamma$$

if Ω is not bordered by grid lines? Discuss this for a five-points star which goes through the boundary.

Is the matrix still symmetric? Does the discrete maximum principle still hold?

Exercise 15:

Let u be the solution to the Poisson equation with Dirichlet boundary conditions:

$$-\Delta u = f \quad \text{in } \Omega\,, \qquad u = g \quad \text{on } \Gamma \tag{$*$}$$

and let $\widetilde{u}$ solve the problem for perturbed boundary data $\widetilde{g}$. Assume $u, \widetilde{u} \in C^2(\Omega) \cap C(\overline{\Omega})$. Show:

$$\sup_{\Omega} |\widetilde{u} - u| \leq \sup_{\Gamma} |\widetilde{g} - g|$$

and further show a similar bound for the finite difference approximation.

Exercise 16:

Let u be the solution to $(*)$ and let $\widetilde{u}$ be the solution to the problem with perturbed right hand side $\widetilde{f}$. Assume $u, \widetilde{u} \in C^2(\Omega) \cap C(\overline{\Omega})$. Show:

$$\sup_{\Omega} |\widetilde{u} - u| \leq \frac{r^2}{4} \cdot \sup_{\Omega} |\widetilde{f} - f|\,,$$

if Ω is contained in a circle of radius r.

Hint: You already know this for the discretized problem.

Programming exercise 3 :

Solve approximately

$$-\Delta u = 1 \quad \text{in } \Omega\,, \qquad u = 0 \quad \text{on } \Gamma$$

using the finite differences method on the unit square Ω - or if you want also on the unit circle. Chose the mesh size $h = 1/16$.

Discussion of the sheet on 27.11.2023.
Hand in the programming exercise by 20.11.2023.