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4. Exercise sheet for numerics of stationary differential equations

Exercise 8:

Suppose there is given a 3-point-boundary value problem of the form

$$
\begin{aligned}
&y' = f(y) \\
&r(y(a), y(\tau), y(b)) = 0, \qquad a < \tau < b.
\end{aligned}
$$

Let $y^\star$ be a solution to the problem. Give a sufficient condition such that $y^\star$ is locally unique.

Hint: Follow the ideas from §2 from the lecture.

Exercise 9:

Formulate the multi shooting method for the 3-point-boundary value problem from exercise 8. How do the linear equation systems, that have to be solved in each Newton-step, look like?

Hint: Suppose that τ is a partition point.

Programming exercise 2 :

Implement the collocation method from exercise 6 ($s = 1$, $c_1 = 1/2$) as a multi shooting method as in exercise 7 for the boundary value problem

$$
y'' + t^{-1}y' - 4t^{-2}y = 0, \qquad y(1) = 2, \quad y(2) = 17/4.
$$

Divide the interval $[1,2]$ equidistant in m sub-intervals $[t_j, t_{j+1}]$, $0 \le j \le m-1$. Test your program for $m = 2^i, 1 \le i \le 6$ and determine $\max_{0 \le j \le m} |u(t_j) - y(t_j)|$, where u is the approximation from the collocation method to the exact solution y. Which convergence rate can you observe? Appropriate initial values for $y(t_j)$ – without any knowledge of the exact solution – can be obtained via linear interpolation (and for $y'(t_j)$ choose simply 0).

Hint: The exact solution of the boundary value problem is $y(t) = t^2 + t^{-2}$.

Discussion of the sheet on 13.11.2023.
Hand in the programming exercise by 20.11.2023.