

11. Exercise sheet for numerics of stationary differential equations

Exercise 30:

Show for a linear function v on a triangle K with diameter h and inner circle radius ρ

$$\|v\|_{\infty} \leq C h^{-1} \|v\|_{0,K},$$

where C is independent of K as long as $h/\rho \leq \text{Const.}$

Note: $\|v\|_{\infty}$ denotes the maximum norm.

Exercise 31:

Let K be a triangle with diameter h and inner circle radius ρ . Show for the interpolation error that it holds

$$\|u - \Pi_h u\|_{\infty} \leq C h |u|_{2,K} \quad \text{for all } u \in H^2(K),$$

where C is independent of K as long as $h/\rho \leq \text{Const.}$

Hint: $H^2(K) \hookrightarrow C(K)$ with $\|\cdot\|_{\infty}$ is continuous and linear according to the Sobolev embedding theorem. Show the statement first for the reference triangle.

Exercise 32:

A H^2 regular boundary value problem is solved with the linear finite elements method. Show for the error that it holds

$$\|u - u_h\|_{\infty} \leq C h |u|_2.$$

Hint: Use $u - u_h = (u - \Pi_h u) + (\Pi_h u - u_h)$, exercises 30 and 31 and then $\Pi_h u - u_h = (\Pi_h u - u) + (u - u_h)$.

Discussion of the sheet on 15.01.2024.

Remember to implement the finite element method.