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Mathematical Institut
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Tübingen, the 16.10.2023

1. Exercise sheet for numerics of stationary differential equations

Exercise 1:
(a) Determine the solution of the 1-dimensional initial value problem

$$y'(t) = \lambda y(t), \quad y(t_0) = y_0 \in \mathbb{R}, \qquad \lambda \in \mathbb{R},$$

for a fixed t_0.

(b) Use this result for the transition to higher dimensions. For this, give the resolvent $R(\cdot,\cdot)$ of the d-dimensional initial value problem

$$y'(t) = Ay(t), \quad y(t_0) = y_0 \in \mathbb{R}^d, \qquad A \in \mathbb{R}^{d\times d}, \tag{$*$}$$

for a fixed t_0 and check your result by inserting it in the differential equation $(*)$.

(c) Can these considerations also be applied to the case of a non-constant $A(t) \in \mathbb{R}^{d\times d}$?

Exercise 2:
Let $R(\cdot,\cdot)$ be the resolvent of the linear differential equation $\tilde{y}'(t) = C(t)\tilde{y}(t)$, with $C(t) \in \mathbb{R}^{d\times d}$. Show:

(a) For fixed t_0, $R(\cdot, t_0)$ is the solution of the problem

$$\frac{\mathrm{d}}{\mathrm{d}t} R(t,t_0) = C(t)R(t,t_0), \qquad R(t_0,t_0) = I.$$

(b) The solution of the inhomogeneous initial value problem

$$y'(t) = C(t)y(t) + q(t), \qquad y(t_0) = y_0 \in \mathbb{R}^d$$

is given by

$$y(t) = R(t,t_0)y_0 + \int_{t_0}^{t} R(t,s)q(s)ds.$$

Discussion of the sheet on 23.10.2023.

Please register in the URM system for the exercises!